correction interrogation

$$A = \begin{pmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \\ 0 & -1 & \alpha \end{pmatrix}$$

1) déterminant de la matrice A :

$$\det A = \begin{vmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \\ 0 & -1 & \alpha \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ -1 & \alpha \end{vmatrix} - 2 \begin{vmatrix} 0 & -1 \\ -1 & \alpha \end{vmatrix} \quad 1$$

$$= \alpha + 2 \quad 1$$

2) Les valeurs α pour la matrice A, soit inversible. 1

A est inversible si $\det A \neq 0$

$$\alpha + 2 \neq 0$$

$$\alpha \neq -2$$

$\alpha \in \mathbb{R} - \{-2\}$ alors A^{-1} existe.

1

3) on calcule la matrice inverse de A si $\alpha = 0$

$$A^{-1} = \frac{1}{\det A} {}^{t}\mathrm{com}\, A \quad 1$$

$\alpha = 0$; $\det A = 2$, $A = \begin{pmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \\ 0 & -1 & 0 \end{pmatrix}$

0,5 0,5

$$\text{com}\, A = \begin{pmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{pmatrix} = \begin{pmatrix} +\begin{vmatrix} 1 & 0 \\ -1 & 0 \end{vmatrix} & -\begin{vmatrix} 2 & 0 \\ 0 & 0 \end{vmatrix} & +\begin{vmatrix} 2 & 1 \\ 0 & -1 \end{vmatrix} \\ -\begin{vmatrix} 0 & -1 \\ -1 & 0 \end{vmatrix} & +\begin{vmatrix} 1 & -1 \\ 0 & 0 \end{vmatrix} & -\begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix} \\ +\begin{vmatrix} 0 & -1 \\ 1 & 0 \end{vmatrix} & -\begin{vmatrix} 1 & -1 \\ 2 & 0 \end{vmatrix} & +\begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} \end{pmatrix}$$

$$C_{11} = (-1)^{1+1} \det A_1^1 = \begin{vmatrix} 1 & 0 \\ -1 & 0 \end{vmatrix} = 0$$

$$C_{12} = (-1)^{1+2} \det A_1^2 = -\begin{vmatrix} 2 & 0 \\ 0 & 0 \end{vmatrix} = 0$$

$$C_{13} = (-1)^{1+3} \det A_1^3 = \begin{vmatrix} 2 & 1 \\ 0 & -1 \end{vmatrix} = -2$$

$$C_{21} = (-1)^{2+1} \det A_2^1 = -\begin{vmatrix} 0 & -1 \\ -1 & 0 \end{vmatrix} = 1$$

$$C_{22} = (-1)^{2+2} \det A_2^2 = \begin{vmatrix} 1 & -1 \\ 0 & 0 \end{vmatrix} = 0$$

$$C_{23} = (-1)^{2+3} \det A_2^3 = -\begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix} = 1$$

$$C_{31} = (-1)^{3+1} \det A_3^1 = \begin{vmatrix} 0 & -1 \\ 1 & 0 \end{vmatrix} = 1$$

$$C_{32} = (-1)^{3+2} \det A_3^2 = -\begin{vmatrix} 1 & -1 \\ 2 & 0 \end{vmatrix} = -2$$

$$C_{33} = (-1)^{3+3} \det A_3^3 = \begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} = 1$$

2

$$\text{com}\, A = \begin{pmatrix} 0 & 0 & -2 \\ 1 & 0 & 1 \\ 1 & -2 & 1 \end{pmatrix}, \quad {}^t\text{com}\, A = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & -2 \\ -2 & 1 & 1 \end{pmatrix}$$

$$A^{-1} = \frac{1}{2} \cdot \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & -2 \\ -2 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & -1 \\ -1 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$ 2

2) $y'' - 4y' + 3y = e^{x}$ ......(*)

la solution est $y(x) = y_H(x) + y_p(x)$

$r^2 - 4r + 3 = 0$ (1) $\Delta = 4$ $\Delta > 0$

$r_1 = 1$ (0,5) $r_2 = 3$ (0,5)

$y_H(x) = c_1 e^{x} + c_2 e^{3x}$ (0,5) où $c_1, c_2 \in \mathbb{R}$.

$\alpha = 1$ est une racine simple de l'équation caractéristique (0,5)

$y_p(x) = xAe^{x}$ (0,5)

$y_p'(x) = Ae^{x} + xAe^{x}$

$y_p''(x) = Ae^{x} + Ae^{x} + xAe^{x}$ on remplace dans (*)

(*): $e^{x}(2A + \cancel{xA} - 4A - 4\cancel{xA} + 3\cancel{xA}) = e^{x}$ (0,5)

$-2A = 1 \Rightarrow A = -\frac{1}{2}$ (0,5)

$y_p(x) = -\frac{1}{2} x e^{x}$ (0,5)

$y(x) = c_1 e^{x} + c_2 e^{3x} - \frac{1}{2} x e^{x}$ (0,5) / $c_1, c_2 \in \mathbb{R}$